B.Tech- 2nd (All Sec.)

Mathematics-II

Full Marks : 50

Time : $2\frac{1}{2}$ hours

Answer **all** questions

The figures in the right-hand margin indicate marks

Symbols carry usual meaning

Any supplementary materials to be provided

1. Answer *all* questions : 2×5

(*a*) Check the exactness of the differential equation $(y\,dy + x\,dx)\cos xy = 0$.

(*b*) Find two linearly independent solutions of $y'' + 2y' + y = 0$.

(*c*) Find the directional derivative of $f(x,y) = 2x^2 + 6xy + z$ at $(9,0,1)$ in the direction of $(1,0,9)$.

(Turn Over)

$z = x+iy$:
$\sin(x+iy)$

$\boxed{\begin{array}{l} \sin(ix) = i\sinh(x) \\ \cos(ix) = \cosh(x) \end{array}}$

(*d*) Find real and imaginary part of $f(z) = z^3$.

(*e*) Find residue of $f(z) = \dfrac{1}{z^2}$ at $z = 0$.

2. (*a*) Solve the ODE $\dfrac{dy}{dx} + \dfrac{y}{x} = \dfrac{y^2}{x}\log(x)$. 4

(*b*) A series RL circuit having a resistance of 20 ohm and inductance of 8H is connected to a DC voltage source of 120V at $t = 0$. Find the current in the circuit at $t = 6$. 4

Or

(*a*) Solve the ODE $(x^2 - xy)dy + (xy - 1)dx = 0$. 4

(*b*) A body at temperature of 40 degree Celsius is kept in a surrounding of constant temperature of 20 degree Celsius. It observed that its temperature falls to 35 degree Celsius in 10 minutes. Find how much

more time will it take for the body to attain a temperature of 30 degree Celsius. 4

3. (*a*) Solve the non-homogeneous ODE

$$y'' + y = \tan(x).$$ 4

(*b*) Find the power series solution of

$$(1 - x^2)y'' - 2xy' + 2y = 0.$$ 4

Or

(*a*) Solve the initial value problem

$$x^2y'' - 3xy' + 4y = 0;\ y(1) = 1,\ y'(1) = 1.$$ 4

(*b*) Find the solution of $y'' - y' + 6y = 0$ using power series method. 4

4. (*a*) Verify Green's theorem for $F = (y^2 - 7y, 2xy + 2x)$ and C the circle with radius 1. 4

(*b*) Prove that for any vector valued function $F(x,y,z)$, $\text{div}(\text{curl}(F)) = 0$. 4

Or

(*a*) Prove that the integral

$$\int_{(-1,7,0)}^{(3,1,9)} \left(xyz^2\,dx + \frac{1}{2}(zx)^2\,dy + x^2\,yz\,dz \right)$$

is path independent in any domain in space and find its value. 4

(*b*) Evaluate $\iint_R (x^2 + y^2)dx\,dy$ over the square R with vertices (1,1), (1,–1), (–1,–1) and (–1,1). 4

5. (*a*) Find real and imaginary part of $\sin z$. 4

(*b*) What is an analytic function ? Test the analyticity of $f(z) = iz + |z|^2$ at $z = 0$. 4

Or

(*a*) Is the function $f(z) = |z|^2$ is differentiable at $z = 0$? If so, find the derivative at $z = 0$. 4

(*b*) Find real and imaginary part of i^{-i}. 4

6. (*a*) Find $\oint_C I_m z\,dz$, where C is a line segment from origin to $1 + i$. 4

(*b*) Find Laurent series of $f(z) = \dfrac{z+1}{z^2+1}$ about $z = i$. 4

Or

(*a*) Integrate the function $f(z) = \dfrac{\sin z}{z^3}$ counter clockwise around the unit circle $|z| = 1$. 4

(*b*) Integrate $f(z) = \dfrac{\sin(z) + 16z}{z^6}$ counterclock-wise around $|z| < 0.5$. 4