

VEER SURENDRA SAI UNIVERSITY OF TECHNOLOGY (VSSUT), ODISHA
Odd Mid-Semester Examination for Academic Session 2025-26

COURSE NAME: BTECH
 BRANCH NAME: ALL

SEMESTER: III

FULL MARKS: 30

SUBJECT NAME: MATH-III

TIME: 90 Minutes

Answer All Questions.

The figures in the right hand margin indicate Marks. *Symbols carry usual meaning.*

- Q1. Answer all Questions. [2 × 3]
- a) Find the Laplace transform of 3^t . - CO1
- b) Find the Fourier coefficients a_0 and a_2 from the Fourier series expansion of $f(x) = \sin^2 x, -\pi < x < \pi$. - CO2
- c) If A and B are independent events then show that A and B^c are also independent. - CO3
- Q2. [4+4]
- a) Find the inverse Laplace transform of $F(s) = \frac{1}{s^2(s^2 - a^2)}$. - CO1
- b) Solve the initial value problem $y'' + 16y = 4\delta(t - \pi)$, $y(0) = 2, y'(0) = 0$ using Laplace transform. - CO1
- OR**
- c) Find the Laplace transform of $f(t) = \begin{cases} \sin t; & 0 < t < \pi \\ t; & t > \pi \end{cases}$. - CO1
- d) Solve the integral equation $y = 1 - \sinh t + \int_0^t (1 + \tau)y(t - \tau) d\tau$ by using convolution. - CO1
- Q3. [4+4]
- a) Find the Fourier series expansion of the function $f(x) = x, 0 < x < 2\pi$. - CO2
- b) Solve the following initial boundary value problem $u_{tt} = c^2 u_{xx}, 0 < x < 1, t > 0$, where $u(x, 0) = x(1 - x), u_t(x, 0) = 0, 0 \leq x \leq 1$ and $u(0, t) = 0 = u(1, t), t > 0$. - CO2
- OR**
- c) Find the Fourier series expansion of the function $f(x) = \begin{cases} x, & 0 < x < 1 \\ 1 - x, & 1 < x < 2 \end{cases}$ with period $P = 2L = 2$. - CO2
- d) By using method of separation of variables solve $3u_x + 2u_y = 0, u(x, 0) = 4e^{-x}$. - CO2
- Q4. [4+4]
- a) A coin is tossed until a head appears. What is the expectation of the number of tosses required? - CO3
- b) If X has the probability density $f(x) = \begin{cases} k.e^{-3x} & \text{for } x > 0 \\ 0 & \text{elsewhere} \end{cases}$. Find k and $P(0.5 \leq X \leq 1)$. - CO3
- OR**
- c) In a Biotechnology Department there are three sections, named as A, B and C. Section A has 15 boys and 10 girls; section B has 20 boys and 25 girls; section C has 10 boys and 20 girls. A section is chosen randomly and a student is selected. What is the probability that the student is a boy? Also find the probability that the boy is from section B. - CO3
- d) Construct a cumulative distribution function for the probability density function $f(x) = kx(1 - x)$ for $0 < x < 1$ and 0, otherwise. - CO3

$L^{-1} \left(\frac{e^{-(t-1)}}{s} \right) = e^{-(t-1)} u(t-1)$