

VEER SURENDRA SAI UNIVERSITY OF TECHNOLOGY (VSSUT), ODISHA

ODD Mid Semester Examination for session 2023-24

B.Tech.(All branches)

1st Semester.

Subject Mathematics-1

Full Mark-30

Time-90 Minutes

Answer All Questions

The figures in the right hand margin indicate Marks. *Symbols carry usual meaning*

1. Answer the following questions.

[2×3]

a). Find the area bounded by the axis of X, the curve $y = \sin^2 x$, $x=0, x=\frac{\pi}{2}$. - CO1

b) Check the convergence of Improper Integral $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$. -CO2

c) Using properties of gamma function find $\beta(5, \frac{1}{2})$. -CO3

2. [4+4]

(a) Find Perimeter of the closed curve $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{a}\right)^{\frac{2}{3}} = 1$. -CO1

(b) State the statement of Dirichlet's Test and use it to check convergence of $\int_0^{\infty} \sin x^2 dx$ -CO1

OR

(c) The Cardioid $r = 5(1 + \cos\theta)$ revolves about initial line ($\theta = 0$). Find the volume of the solid thus generated. -CO1

(d) Calculate area of the loop of the curve $a^2 y^2 = x^3(2a - x)$. -CO1

3. [4+4]

(a) Discusses different cases of convergences and divergences depending various values of real number n of the integral $I = \int_a^{\infty} \frac{dx}{x^n}$, $a > 0$ -CO2

(b) Write steps of First Derivative Test to find local maximum or local minimum and write nature of various critical points of the given real valued function $f(x) = (x - 2)^4(x + 1)^3$ -CO2

OR

(c) Discuss difference and similarity between Abel's Test and Dirichlet's Test.

Show that the improper integral $\int_1^{\infty} \frac{dx}{(1+x)\sqrt{x}}$ is convergent. -CO2

(d) Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. -CO2

4. [4+4]

(a) State mean value theorem and use it to find a point of the curve $y = \cos x - 1$, $x \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ at which tangent is parallel to X-axis -CO3

(b) Find Maclaurin's series of $f(x) = 3x^4 + x^3 - 10x^2 + 6$. -CO3

OR

(c) Find Taylor's Series of $f(x) = e^{-x^2 + \cos x}$ about $x = \frac{\pi}{2}$. -CO3

(d) Verify mean value theorem for the function $f(x) = (x - 3)(x - 6)(x - 9)$ on the interval $[3, 5]$ -CO3